THE USE OF ELECTRE I METHOD IN THE DECISION-MAKING PROCESS - CASE STUDY

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Abstract:	This article presents the possibilities of using one of the multi-criteria discrete methods in the engineering design. Illustrated with an example, the way of using Electre I method in the evaluation of quality features, which describe technical means the purchase of which is being taken into consideration in connection with the expected start-up of a new coal-mine area, has been discussed.
Keywords:	Statistical methods, Discrete methods, Multi-criteria methods, Decision theory, Electre I.

Introduction

In the selection of technical and organisational system, apart from single-criteria evaluations (the evaluation criterion is, for example, the limit payback period), more and more often the multi-criteria evaluations are also used. The example of the latter ones includes, but is not limited to, the distance measure formulas and multi-criteria discrete methods (Mynarski, 1992; Trzaskalik, 2006). For methods using the distance measure formulas there can be distinguished, among other things, the Hamming, Euclidean, Bray-Curtis, Jeffreys-Matusita, Clark and Canberra distances, while for multi-criteria discrete methods - the Analytic Hierarchy Process (AHP), Promethee II and Electre I methods. The last method allows the set of n variants of decision to be analysed and evaluated with regard to k maximised f_k criteria - each individual variant is assigned by the decision maker the importance w_k ($\sum\limits_{i=1}^{k} w_i = 1$) (Hillier and Lieberman, 1990; Trzaskalik, 2001; Wagner, 1980). With two variants of decision x and y, the decision maker determines the outranking index $\varphi_i(x, y)$:

$$\varphi_i(x, y) = \begin{cases} 1, & \text{when } f_i(x) \geq f_i(y) \\ 0, & \text{otherwise} \end{cases}$$

which determines whether variant x is evaluated better or worse than variant y with regard to consecutive f_k criteria. Then, in order to determine the degree of predominance of variant x over variant y, the concordance index $c(x, y)$ is calculated:

$$c(x, y) = \sum\limits_{i=1}^{k} w_i \varphi_i(x, y)$$

which is the sum of weights of these criteria for which the values of variant x are not smaller than those for variant y. The concordance condition is assumed to be met when the concordance index is not lower than the indifference threshold value adopted by the decision maker (s):

$$c(x, y) \geq s, \quad s \in \langle 0.5; 1.0 \rangle$$

In order to eliminate these cases where the concordance condition is admittedly met, but at least one of the criteria of predominating variant x has unfavourable value, the lack of concordance condition is checked (the decision maker adopts subjectively the so-called veto threshold value).

The decision-making procedure using Electre I method ends with outlining the graph for relationship between variants of decision (Hillier and Lieberman, 1990; Trzaskalik, 2006; Trzaskalik, 2001; Wagner, 1980).

Materials and Methods

The example of the use of Electre I method in the decision-making process related to purchase of the means of transport

Start-up of exploitation in a new coal-mine area is expected, therefore the purchase of the means of

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transport for carrying people and materials to the said area is being considered. As a part of the initial evaluation (after technical parameters of the transport machinery and equipment were analysed and initial calculation of the costs related to the purchase/ use of individual variants of technical equipment was carried out), three alternatives were selected: purchase of the cog-wheel railway (variant a^1), purchase of the suspended diesel railway (variant a^2) and purchase of the suspended railway with rope drive (variant a^3). In addition, six evaluation criteria were defined: ease of assembly (f_1); availability of spare parts (f_2); safety (f_3); potential for expansion (f_4); terms and conditions of warranty and service (f_5) and manufacturer's brand (f_6).

After survey research in the group of experts (employees of the materials management department, supervisory personnel of the electrical power and mechanical department and employees of the underground transport department), the consecutive criteria were assigned the importance coefficients:

$$w_1 = 0.3$$
$$w_2 = 0.2$$
$$w_3 = 0.2$$
$$w_4 = 0.1$$
$$w_5 = 0.1$$
$$w_6 = 0.1$$

whereas $\sum_{i=1}^{6} w_i = 1$

Tab. 1 presents the evaluations of variants taken into consideration with regard to the defined criteria (the evaluation scale for each of the criteria from 0 to 10).

Tab. 1 Evaluation of equipment variants with regard to the adopted criteria

		Criterion					
		f_1	f_2	f_3	f_4	f_5	f_6
Variants	a_1	7	7	10	3	4	2
	a_2	3	9	10	1	7	9
	a_3	3	6	10	8	6	3

For consecutive criteria the following veto thresholds were adopted:

$$v_1[f_1(a^i)] = 4 \quad v_4[f_4(a^i)] = 7$$
$$v_2[f_2(a^i)] = 6 \quad v_5[f_5(a^i)] = 6$$
$$v_3[f_3(a^i)] = 6 \quad v_6[f_6(a^i)] = 2$$

At the same time, the indifference threshold of 0.6 ($s = 0.6$) was adopted.

In the first stage, the set of concordances was determined - for $k \in \langle 1; 6 \rangle$ of possible pairs of variants of decision and the values of $\varphi_k(a^i, a^j)$ were calculated. The results are summarised in Tab. 2.

Tab. 2 Sets of concordances for pairs of variants

Φ_1	a^1	a^2	a^3
a^1	1	1	1
a^2	0	1	1
a^3	0	1	1

Φ_2	a^1	a^2	a^3
a^1	1	0	1
a^2	1	1	1
a^3	0	0	1

Φ_3	a^1	a^2	a^3
a^1	1	1	1
a^2	1	1	1
a^3	1	1	1

Φ_4	a^1	a^2	a^3
a^1	1	1	0
a^2	0	1	0
a^3	1	1	1

Φ_5	a^1	a^2	a^3
a^1	1	0	0
a^2	1	1	1
a^3	1	0	1

Φ_6	a^1	a^2	a^3
a^1	1	0	0
a^2	1	1	1
a^3	1	0	1

Based on the adopted importance coefficients w_k and values of $\varphi_k(a^i, a^j)$, the concordance indices were determined:

$$c(a^1, a^2) = w_1\varphi_1(a^1, a^2) + w_2\varphi_2(a^1, a^2) +$$
$$+ w_3\varphi_3(a^1, a^2) + w_4\varphi_4(a^1, a^2) +$$
$$+ w_5\varphi_5(a^1, a^2) + w_6\varphi_6(a^1, a^2) = 0.6$$

$$c(a^1, a^3) = w_1\varphi_1(a^1, a^3) + w_2\varphi_2(a^1, a^3) +$$
$$+ w_3\varphi_3(a^1, a^3) + w_4\varphi_4(a^1, a^3) +$$
$$+ w_5\varphi_5(a^1, a^3) + w_6\varphi_6(a^1, a^3) = 0.7$$

$$c(a^2, a^1) = w_1\varphi_1(a^2, a^1) + w_2\varphi_2(a^2, a^1) +$$
$$+ w_3\varphi_3(a^2, a^1) + w_4\varphi_4(a^2, a^1) +$$
$$+ w_5\varphi_5(a^2, a^1) + w_6\varphi_6(a^2, a^1) = 0.6$$

$$c(a^2, a^3) = w_1\varphi_1(a^2, a^3) + w_2\varphi_2(a^2, a^3) +$$
$$+ w_3\varphi_3(a^2, a^3) + w_4\varphi_4(a^2, a^3) +$$
$$+ w_5\varphi_5(a^2, a^3) + w_6\varphi_6(a^2, a^3) = 0.9$$

$$c(a^3, a^1) = w_1\varphi_1(a^3, a^1) + w_2\varphi_2(a^3, a^1) +$$
$$+ w_3\varphi_3(a^3, a^1) + w_4\varphi_4(a^3, a^1) +$$
$$+ w_5\varphi_5(a^3, a^1) + w_6\varphi_6(a^3, a^1) = 0.5$$

$$c(a^3, a^2) = w_1 \varphi_1(a^3, a^2) + w_2 \varphi_2(a^3, a^2) +$$
$$+ w_3 \varphi_3(a^3, a^2) + w_4 \varphi_4(a^3, a^2) +$$
$$+ w_5 \varphi_5(a^3, a^2) + w_6 \varphi_6(a^3, a^2) = 0.6$$

The results of calculation are summarised in Tab. 3.

Tab. 3 Results of calculation

C	a^1	a^2	a^3
a^1	1	0.6	0.7
a^2	0.6	1	0.9
a^3	0.5	0.6	1

The set of concordances in the binary form (for the adopted indifference threshold $s = 0.6$) is presented in Tab. 4.

Tab. 4 Set of concordances in the form of binary matrix

C	a^1	a^2	a^3
a^1	1	1	1
a^2	1	1	1
a^3	0	1	1

Results

For pairs of variants for which the concordance condition has been met, the lack of disconcordance condition was checked then:

for $k = 1$

$(a^1, a^2): f_1(a^1) + v_1\left[f_1(a^1)\right] \geq f_1(a^2)$ $(7 + 4 \geq 3)$

no disconcordance

$(a^1, a^3): f_1(a^1) + v_1\left[f_1(a^1)\right] \geq f_1(a^3)$ $(7 + 4 \geq 3)$

no disconcordance

$(a^2, a^1): f_1(a^2) + v_1\left[f_1(a^2)\right] \geq f_1(a^1)$ $(3 + 4 \geq 7)$

no disconcordance

$(a^2, a^3): f_1(a^2) + v_1\left[f_1(a^2)\right] \geq f_1(a^3)$ $(3 + 4 \geq 3)$

no disconcordance

$(a^3, a^2): f_1(a^3) + v_1\left[f_1(a^3)\right] \geq f_1(a^2)$ $(3 + 4 \geq 3)$

no disconcordance

for $k = 2$

$(a^1, a^2): f_2(a^1) + v_2\left[f_2(a^1)\right] \geq f_2(a^2)$ $(7 + 6 \geq 9)$

no disconcordance

$(a^1, a^3): f_2(a^1) + v_2\left[f_2(a^1)\right] \geq f_2(a^3)$ $(7 + 6 \geq 6)$

no disconcordance

$(a^2, a^1): f_2(a^2) + v_2\left[f_2(a^2)\right] \geq f_2(a^1)$ $(9 + 6 \geq 7)$

no disconcordance

$(a^2, a^3): f_2(a^2) + v_2\left[f_2(a^2)\right] \geq f_2(a^3)$ $(9 + 6 \geq 6)$

no disconcordance

$(a^3, a^2): f_2(a^3) + v_2\left[f_2(a^3)\right] \geq f_2(a^2)$ $(6 + 6 \geq 9)$

no disconcordance

for $k = 3$

$(a^1, a^2): f_3(a^1) + v_3\left[f_3(a^1)\right] \geq f_3(a^2)$ $(10 + 6 \geq 10)$

no disconcordance

$(a^1, a^3): f_3(a^1) + v_3\left[f_3(a^1)\right] \geq f_3(a^3)$ $(10 + 6 \geq 10)$

no disconcordance

$(a^2, a^1): f_3(a^2) + v_3\left[f_3(a^2)\right] \geq f_3(a^1)$ $(10 + 6 \geq 10)$

no disconcordance

$(a^2, a^3): f_3(a^2) + v_3\left[f_3(a^2)\right] \geq f_3(a^3)$ $(10 + 6 \geq 10)$

no disconcordance

$(a^3, a^2): f_3(a^3) + v_3\left[f_3(a^3)\right] \geq f_3(a^2)$ $(10 + 6 \geq 10)$

no disconcordance

for $k = 4$

$(a^1, a^2): f_4(a^1) + v_4\left[f_4(a^1)\right] \geq f_4(a^2)$ $(3 + 7 \geq 1)$

no disconcordance

$(a^1, a^3): f_4(a^1) + v_4\left[f_4(a^1)\right] \geq f_4(a^3)$ $(3 + 7 \geq 8)$

no disconcordance

$(a^2, a^1): f_4(a^2) + v_4\left[f_4(a^2)\right] \geq f_4(a^1)$ $(1 + 7 \geq 3)$

no disconcordance

$(a^2, a^3): f_4(a^2) + v_4\left[f_4(a^2)\right] \geq f_4(a^3)$ $(1 + 7 \geq 8)$

no disconcordance

$(a^3, a^2): f_4(a^3) + v_4\left[f_4(a^3)\right] \geq f_4(a^2)$ $(8 + 7 \geq 1)$

no disconcordance

for $k = 5$

$(a^1, a^2): f_5(a^1) + v_5\left[f_5(a^1)\right] \geq f_5(a^2)$ $(4 + 6 \geq 7)$

no disconcordance

$(a^1, a^3) : f_5(a^1) + v_5 \left[f_5(a^1) \right] \geq f_5(a^3) \qquad (4 + 6 \geq 6)$

no disconcordance

$(a^2, a^1) : f_5(a^2) + v_5 \left[f_5(a^2) \right] \geq f_5(a^1) \qquad (7 + 6 \geq 4)$

no disconcordance

$(a^2, a^3) : f_5(a^2) + v_5 \left[f_5(a^2) \right] \geq f_5(a^3) \qquad (7 + 6 \geq 6)$

no disconcordance

$(a^3, a^2) : f_5(a^3) + v_5 \left[f_5(a^3) \right] \geq f_5(a^2) \qquad (6 + 6 \geq 7)$

no disconcordance

for $k = 6$

$(a^1, a^2) : f_6(a^1) + v_6 \left[f_6(a^1) \right] \geq f_6(a^2) \qquad (2 + 2 \geq 9)$

no disconcordance

$(a^1, a^3) : f_6(a^1) + v_6 \left[f_6(a^1) \right] \geq f_6(a^3) \qquad (2 + 2 \geq 3)$

no disconcordance

$(a^2, a^1) : f_6(a^2) + v_6 \left[f_6(a^2) \right] \geq f_6(a^1) \qquad (9 + 2 \geq 2)$

no disconcordance

$(a^2, a^3) : f_6(a^2) + v_6 \left[f_6(a^2) \right] \geq f_6(a^3) \qquad (9 + 2 \geq 3)$

no disconcordance

$(a^3, a^2) : f_6(a^3) + v_6 \left[f_6(a^3) \right] \geq f_6(a^2) \qquad (3 + 2 \geq 9)$

no disconcordance

The results of calculation are summarised in Tab. 5.

Tab. 5 Results of calculation

	a¹	a²	a³			a¹	a²	a³
a¹	*	0	0		a¹	*	0	0
a²	0	*	0		a²	0	*	0
a³	*	0	*		a³	*	0	*

	a¹	a²	a³			a¹	a²	a³
a¹	*	0	0		a¹	*	0	0
a²	0	*	0		a²	0	*	0
a³	*	0	*		a³	*	0	*

	a¹	a²	a³			a¹	a²	a³
a¹	*	0	0		a¹	*	1	0
a²	0	*	0		a²	0	*	0
a³	*	0	*		a³	*	1	*

Thus, the set of disconcordances D_v:

$$D_v = \left\{ \left(a^1, a^2 \right), \left(a^3, a^2 \right) \right\}$$

in the binary form is presented in Tab. 6.

Tab. 6 Set of disconcordances in the form of binary matrix

	a¹	a²	a³
a¹	0	1	0
a²	0	0	0
a³	0	1	0

The set of complements for the set of disconcordances D_v is presented in Tab. 7.

Tab. 7 Set of complements for the set of disconcordances

	a¹	a²	a³
a¹	1	0	1
a²	1	1	1
a³	1	0	1

Based on Tab. 4 and Tab. 7, the outranking relations are determined (Tab. 8) - the relation occurs when both the concordance condition (Tab. 4) and the lack of concordance condition (Tab. 7) are met:

$$S(s,v) = C_s \cap D_v = \left\{ (a^1, a^3), (a^2, a^1), (a^2, a^3) \right\}$$

Tab. 8 Outranking relation for variants a^1, a^2, a^3

	a¹	a²	a³
a¹	1	0	1
a²	1	1	1
a³	0	0	1

Graphic image of relationships between variants is presented in the graph in Fig. 1.
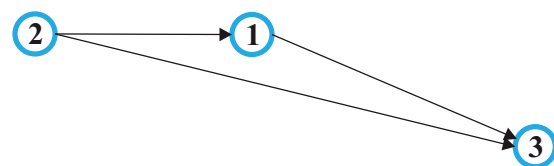

Fig. 1 Graphic image of relationships between variants

Thus, best evaluated is variant no. 2, i.e. purchase of the suspended diesel railway - variant no. 2 is in outranking relation to both variant no. 1 (purchase of the cog-wheel railway) and variant no. 3 (purchase of the suspended railway with rope drive).

Conclusion

In the broadly defined management process, among other things the operational research methods

where the intended objective accomplishment degree is measured by the function of objective (function of criterion) are used very often, and the complexity of the outside world makes the multi-criteria tasks predominate. The Electre I method presented in this article is one of the so-called multi-criteria discrete methods used in relation to the quality features. In this method, the decision maker determines the finite set of variants of decision (in this example, three alternative solutions for purchase of the means of transport are taken into consideration) and the finite set of criteria (in this case, comprised of 6 elements), which allows generating a single variant that best corresponds to its preferences.

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