STOCHASTIC METHODS IN RISK ANALYSIS

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Review article

Abstract: In this paper, we review basic stochastic methods which can be used to extend state-of-the-art deterministic analytical methods for risk analysis. We can conclude that the standard deterministic analytical methods highly depend on the practical experience and knowledge of the evaluator and therefore, the stochastic methods should be introduced. The new risk analysis methods should consider the uncertainties in input values. We present how large is the impact on the results of the analysis solving practical example of FMECA with uncertainties modelled using Monte Carlo sampling.

Keywords: Analytical methods, stochastics methods, risk analysis, safety management, Monte Carlo.

Introduction

During the last decades, the importance of safety is increasing. There are several reasons; for instance, the extraordinary events which cannot be predicted in general, as well as more responsible and fairer attitude of the organizations to safety and security. Moreover, since the new technologies are coming every day, the arising new questions in the terms of safety are much more sophisticated than any time before. Besides these complications, the new methods for risk analysis have to be developed. This paper presents the quick overview of state-of-the-art deterministic and possible stochastic approaches for general risk analysis.

Risk management vs Risk analysis methods

What is Risk Management? Risk is part of all our lives. As a society, we need to take risks to grow and develop. From energy to infrastructure, supply chains to airport security, hospitals to housing, effectively managed risks help societies achieve. In our fast-paced world, the risks we have to manage evolve quickly. We need to make sure we manage risks so that we minimise the threats and maximise their potential.

Risk management is the process of identifying, analysing and responding to risk factors throughout the life of a project and in the best interests of its objectives. Proper risk management implies control of possible future events and is proactive rather than reactive (Stanleigh, 2016).

Risk management involves understanding, analysing and addressing risk to make sure organisations achieve their objectives (IRM, 2016). Fig. 2 presents the cycle of risk management processes as well as the basic idea of generally all methods.

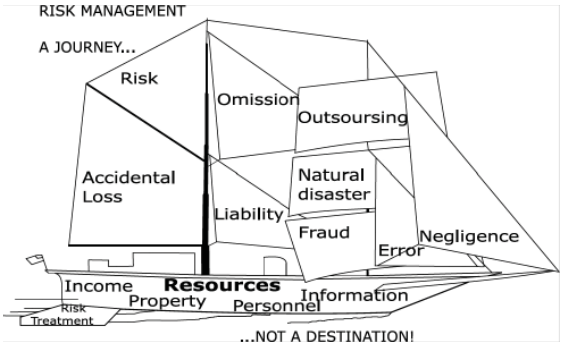

Fig. 1 Risk management (Knight, 2010)

Fig. 2 Planning for risk management (Rahim, 2016)

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The problematics of risk management is extensive, complex, with large number of individual elements and connections between them. Therefore, there is no way how to formulate a general method for all applications.

The selection of the most appropriate method depends on the several criteria, for instance the content and the objectives of risk analysis, environment in which risks occur, as well as quality and quantity of information available for analysis (Kráľová, 2006), (Uniza, 2015). In the most cases, the combination of several methods comes into play, however, some of them cannot be combined because of contradictory assumptions and requirements. The most challenging risk analysis methods are typically based on the analysis of historical dataset of events of interest in the past to understand the inner processes which lead to occurrence of these events. Only after understanding all causalities in given historical data, we are able to estimate the probability of occurrence of these events in the future (i.e., forecast).

However, sometimes the risk analysis suffers from the lack of input data. In this case, we have to rely on the expert analysis. The aim of the modern method is to avoid (or take into account) the subjectivity of these data.

Deterministic methods in risk assessment

In this Section, we present a short overview of the most common state-of-the-art deterministic methods for risk analysis. Nowadays, these methods are used by experts all over the world to analyse and predict the risk based on the subjective expert experiences.

Tab. 1 The list of basic deterministic method for risk analysis

Methods	Definition
Event Tree Analysis (ETA)	Graphical technique that uses Boolean operators to evaluate the consequences of a risk by drawing (mapping) all probable outcomes of an initiating event in their logical sequence (dictionary, 2017).
Fault tree analysis (FTA)	FTA is a top down, deductive failure analysis in which an undesired state of a system is analysed using Boolean logic to combine a series of lower-level events. This analysis method is mainly used in the fields of safety engineering and reliability engineering to understand how systems can fail, to identify the best ways to reduce risk or to determine (or get a feeling for) event rates of a safety accident or a system level (functional) failure (1dictionary, 2017).
Failure Mode and Effect Analysis (FMEA)	FMEA is a structured approach for discovering potential failures that may exist within the design of a product or process. Failure modes are the ways in which a process can fail. Effects are the ways that these failures can lead to waste, defects or harmful outcomes for the customer. Failure Mode and Effects Analysis is designed to identify, prioritize and limit these failure modes (1dictionary, 2017).
Failure Mode, Effect and Critical Analysis (FMECA)	Procedure that follows FMEA, and where each potential failure effect is classified according to its probability of occurrence and degree of severity (dictionary, 2017).
What If Analysis (WI)	Which key quantitative assumptions and computations (underlying a decision, estimate, or project) are changed systematically to assess their effect on the final outcome. Employed commonly in evaluation of the overall risk or in identification of critical factors, it attempts to predict alternative outcomes of the same course of action. In comparison, contingency analysis uses qualitative assumptions to paint different scenarios (dictionary, 2017).
Preliminary Hazard Analysis (PHA)	It used at the beginning of the analytical process for identification of sources of risk. The result is a qualitative description and sequence sources of risk. It is also used to identify hazards in primary, design phase of the project, before it is determined by the final draft project. Its purpose is to identify the design modifications, which would limit or eliminate hazards, and / or mitigate the consequences of accidents (VUBP, 2004).
Cause - Consequence Analysis (CCA)	The combination method (FTA-ETA), based on the probabilistic approach, which examines the early relevant events and the development of the final state accidents with respect to their causes (VUBP, 2004).
Point method	The size (scale) of the risk is a combination of the probability of occurrence of risk and potential severity of the consequences of risk. The risk is always referenced to the work position and work place. Protected value is human life and health (dictionary, 2017).

Stochastic methods in risk assessment

Let us remind that the deterministic methods listed in previous section are based mostly on subjective experiences and on the deterministic input. If we are interested in the methods which can deal with uncertainties in the input data, then we are talking about stochastic methods. These methods are using probabilistic analysis and they provide an opportunity to include the uncertainties in input caused by subjective influence of human expertise.

Nowadays, the stochastic methods are used for predicting probability processes, such as time-series analysis (for instance weather and environment forecasting, risk in economics and predictions of financial markets, etc.). It seems that the next generation of risk analysis methods can be constructed as an extension of the deterministic methods by these well-known stochastic approaches.

Let us notice that there are already developed methods which combines the deterministic and stochastic methods. For example, in the case of ETA and FTA, which fundamentally include logical operators, these operators can be extended by their stochastic variants based on conditional probabilities.

Obviously, our list in Tab. 1 is not complete. We decide to mention only the basic variants of the methods, there exist other methods based on the combination of these primal methods.

Tab. 2 Basic stochastic methods

Stochastic Methods	Description of the method
Monte Carlo	Monte Carlo are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. Their essential idea is using randomness to solve problems that might be deterministic in principle (1dictionary, 2017).
Time-series stochastic processes/ time-series methods (TSM)	Data with a pattern ("trend") over time. Time-series methods make forecasts based solely on historical patterns in the data. Time-series methods use time as independent variable to produce demand (dictionary, 2017).
Markov chain analysis	Sequence of stochastic events (based on probabilities instead of certainties) where the current state of a variable or system is independent of all past states, except the current (present) state (dictionary, 2017).
Scenario analysis	Scenario analysis is a process of analysing possible future events by considering alternative possible outcomes (sometimes called "alternative worlds"). Thus, scenario analysis, which is one of the main forms of projection, does not try to show one exact picture of the future. Instead, it presents several alternative future developments. Consequently, a scope of possible future outcomes is observable (1dictionary, 2017).
Regression method/analysis	Regression method is widely used for prediction and forecasting, where its use has substantial overlap with the field of machine learning. Regression analysis is also used to understand which among the independent variables are related to the dependent variable, and to explore the forms of these relationships (1dictionary, 2017).
Neural networks	Neural networks mimic the operation of the human brain (nerves and neurons). Each neural unit is connected with many others, and links can be enforcing or inhibitory in their effect on the activation state of connected neural units. Each individual neural unit may have a summation function which combines the values of all its inputs together. There may be a threshold function or limiting function on each connection and on the unit, itself, such that the signal must surpass the limit before propagating to other neurons. These systems are self-learning and trained, rather than explicitly programmed, and excel in areas where the solution or feature detection is difficult to express in a traditional computer program (1dictionary, 2017), (dictionary, 2017).
Bayesian networks	Bayesian networks is a model. It reflects the states of some part of a world that is being modelled and it describes how those states are related by probabilities. The model might be of your house, or your car, your body, your community, an ecosystem, a stock-market, etc. Absolutely anything can be modelled by a Bayes net. All the possible states of the model represent all the possible worlds that can exist, that is, all the possible ways that the parts or states can be configured (1dictionary, 2017), (dictionary, 2017).

Materials and methods

In this paper, we demonstrate the possibility of using stochastic methods in risk analysis on the simple example. We take FMECA method and simulate uncertainties in input data of this method using Monte Carlo sampling. In our toy example, the provided benchmark data origin from project SP2016 / 105. However, in this case we simplified them and provide only a part. Such a small number of data will better demonstrate the efficiency of our approach. Therefore, the results cannot be considered as a complete analysis of real-world example.

However, performing calculations with distributions is not easy as it is often not possible to derive analytical solutions unless the distributions have well-specified shapes, and then only with restrictions and assumptions that might not be realistic. In these circumstances, techniques such as Monte Carlo simulation provide a way of undertaking the calculations and developing results.

In general, Monte Carlo simulation can be applied to any system for which:

- a set of inputs interact to define an output;
- the relationship between the inputs and outputs can be expressed as logical and algebraic relationships;
- analytical techniques are not able to provide relevant results or when there is uncertainty in the input data.

An analysis of the relationships between inputs and outputs can throw light on the relative significance of the uncertainty in input values and identify targets for efforts to influence the uncertainty in the outcome (ISO 31010), (Greenland, 2001), (Montgomery, 2009).

FMECA with uncertainties in the input data simulated by Monte Carlo method

To demonstrate the influence of subjective ratings onto the results of derministic analytical method, let us consider a simple example of standard FMECA method. In the Tab. 4, we present a list of possible risks in the small administrative company. FMECA method is based on rating these possible risks using the value of risk priority number (*RPN*), which is given as a product of severity (*S*), occurrence (*O*), and detection (*D*) of appropriate possible risk

$$RPN = S \cdot O \cdot D \quad \text{[CSN EN 60812]}$$

The values of these three properties are assigned as a number 1,2,3,4, or 5 based on expertise and experiences of safety engineer, see Tab. 3.

Tab. 3 FMECA - Value matrix

Severity (*S*)	**Occurrence (*O*)**	**Detection (*D*)**	**Point**
Unlikely	Harm health	Be negligible	1
Random	Injuries resulting in injury	Little impact	2
Likely	Serious accident	Negligible impact	3
Very likely	Heavy industrial accident permanent consequences	Significant	4
Permanent	Fatal accident at work	More significant effects	5

Tab. 4 FMECA - values from risk analysis in the small administrative company

ID	**Hazard agent**	**Source risk**	**Result**	**Risk**			
				S	**O**	**D**	**RPN**
1	Stairs	damaged stairs	repair	2	3	2	12
2	Floor	wet floor	tripping, fall of	3	2	3	18
3	Microclimate	Inadequate	Headache, nausea	3	2	4	24
4	Space workplace	Insufficient workspace	Neck and back problems	3	1	2	6
5	Team	Bullying in the workplace	Stress	1	2	4	8
6	Doors	Opening doors, open to the corridor	Crushing, bruises pinching	3	1	3	9
7	Windows	Sun shining/ unwary opening	Damage to sight/ fall of	2	2	1	4
8	Illumination	unsuitable Lighting	Damage to sight	1	2	5	10
9	Electrical equipment	Damaged electrical devices, switches, sockets	Electric shock	4	2	3	24
10	Evacuations corridors	Blocked evacuations corridors	Injury or dead	1	5	1	5
11	Work on PC	Long - term stress under adverse conditions, the optical	Visual impairment	5	2	3	30

From Tab. 4, we can see that the riskiest (based on the *RPN* number) is working on PC. The company should pay the most attention to decrease appropriate *RPN*, i.e. decrease one (or more than one) of the values of *S*, *O*, or *D*.

Let us consider a situation, when this evaluation of the expert is not exact, i.e. there are some misclassified values of S, O, or D. To determine the stability of provided maximum risk, let us consider a small perturbation in input values presented in Tab. 4 implementing the additional Gaussian noise. The new values of these random values are given by:

$$\tilde{S} = S + \varepsilon_S,$$
$$\tilde{O} = O + \varepsilon_O,$$
$$\tilde{D} = D + \varepsilon_D,$$
$$\varepsilon_S, \varepsilon_O, \varepsilon_D \sim N(0, 0.5)$$

and corresponding RPN number is now also a random variable:

$$\widetilde{RPN} = \tilde{S} \cdot \tilde{O} \cdot \tilde{D}$$

Let us notice that even when three input values $\tilde{S}$, $\tilde{O}$, $\tilde{D}$ are normally distributed, the product of these tree numbers $\widetilde{RPN}$ is not normally distributed (Springer & Thompson, 1970). To simulate the randomness, we implement a simple code in Matlab, where we generated 10^6 sample values of $\tilde{S}$, $\tilde{O}$, $\tilde{D}$ and for each triplet of these values, we compute appropriate $\widetilde{RPN}$s (for every risk scenario). Afterwards, for every random sample, we sort the risk scenarios in descent order. Finally, we count the numbers of order through samples for every risk.

Results

The figure demonstrates the stability of the order of given risk scenarios with uncertainties in the data. We can see that still the riskiest is 11th scenario, however, the 9th scenario and 3rd scenarios are also risky. For instance, the 3rd and 9th scenario in certain cases after the perturbation of the input values are located on the first position (the riskiest scenario). Furthermore, if we increase the standard deviation of normally distributed noise to the value of 1.5, then the uncertainty of the final order is even more increased (see Fig. 4).

To compare the method with standard deterministic FMECA method, we reduced the standard deviation of normally distributed noise to the value 0.05 in the last example, see Fig. 5. We can observe that with this small number, the final order is almost stable. Therefore, we can conclude that with decreasing deviation of error the results converge to the standard deterministic FMECA method.

This simple example demonstrates how the subjective evaluation in risk analysis influences the final risk prioritization.

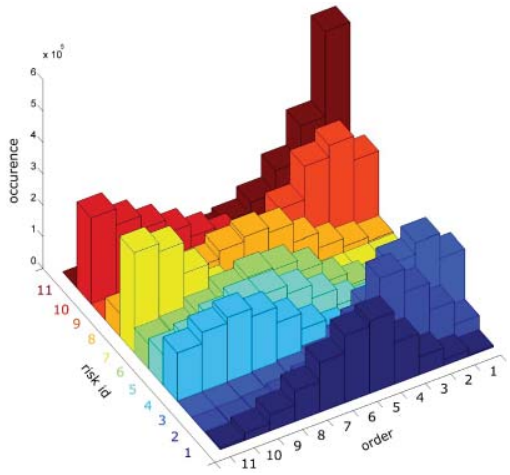

Fig. 3 Input values (0.5) with small perturbation (author)

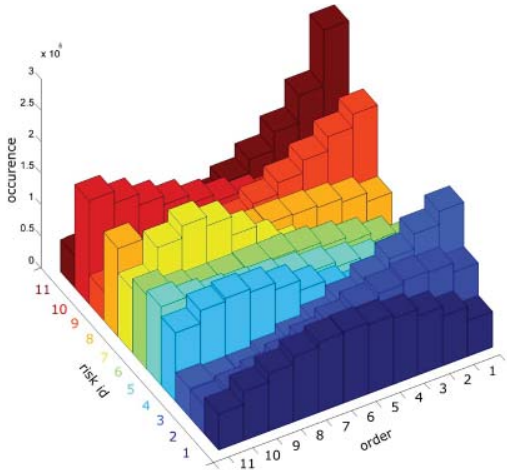

Fig. 4 Input values (1.5) with higher perturbation (author)

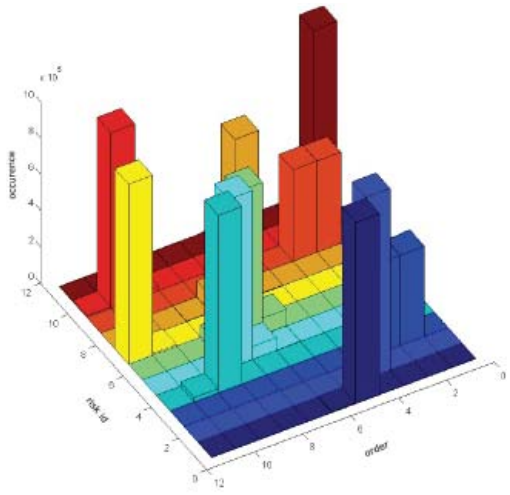

Fig. 5 Input values with smaller perturbation (author)

Conclusion

We introduced a list of basic analytical and stochastic methods for risk analysis. Not all of them are suitable for combining with each other. As one good candidate appears to combination methods is analytical FMEA and stochastic DEA. This approach has been published in (Osadská et. al, 2017).

Other good candidates are analytical methods with point assessment which are promising to be combined with Monte Carlo simulation. One of these combination (FMECA) has been presented in this paper. However, the Safety is complex area which includes many sub-regions and the choice of the risk analysis method depends on many factors.

In this paper, we demonstrated the influence of subjective input into the result of risk analysis method. In our opinion, this influence should be always considered and modern methods should implement the stochastic approach to deal with the uncertainties in input data.

We demonstrated the influence of uncertainties in input data on the example of FMECA. In this case, we simulated the randomness in the data using randomly distributed values. We show that the standard deviation of chosen distribution influences the final results and stability of the method.

This is the reason, why we will focus on practical application of methods with stochastic approach in our future work.

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